***Second Annual NCMATYC Math Tournament***

***November 19, 2011***

***Morning Component***

***Good morning!***

***Please do NOT open this booklet until given the signal to begin.***

***There are 40 multiple choice questions and you will be given 90 minutes to complete the test. Answer the questions on the electronic grading form by giving the best answer to each question.***

***The scoring will be done by giving one point for each question answered correctly and zero points for each question answered incorrectly or left blank. Thus, it is to your advantage to answer as many questions as possible, even if you have to guess. If there is a tie, question number 35 will be used as a tie-breaker.***

***This test was designed to be a CHALLENGE. Do not waste time on questions you are unable to answer; focus and take pride in those questions which you ARE able to answer.***

***You may write in the test booklet. You may keep your test booklet and any of your scrap papers. Only the electronic grading form will be collected and graded.***

***Good luck!***

***Do Not Open Until Signaled.***

1. Let *f*(*x*) be a one to one function with and for all . Which of the following is true?

A. B. C. D. E.

1. If. On which of the following intervals is *f* increasing?

A. B. C. D. E.

1. Assume , compute

A. B. C. D. 0 E. undefined

1. If , then which expression is equivalent to ?

A. B. C**.**D**.** E.

1. What is 98th derivative of the function ?

A. B. C.D. E. none of these

1. What is ?

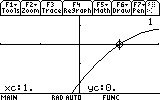
A. 0 B. 2 C. 1 D. E. does not exist

1. If , then which expression is equivalent to ?

A. B. C.  
  
D. E.

1. Compute .

A. B. C. D. E. none of these



1. The graph displayed to the right represents which of the following is true?

A. B. C.   
  
 D. E.

1. Let *f* (*x*) be continuous and differentiable on and let . Given that the maximum value of *f* is 9 and it occurs when *x* = 3, then what is the value of ?

A. B. C. D. E. 3

1. Which of the following functions is differentiable at *x* = 0?

A. *y = * B. *y =* C. *y =* D. *y =* E. none of these

1. Evaluate .

A. B. C. D. E. none of these

1. For any positive integer *n*, determine  for .

A. *n*! B. (*n* + 1)! C. (*x*)(*n*!) D. (*x*)[(*n* + 1)!] E. none of these

1. At how many points does the slope of  equal 1?

A. B. C. D. E. 4

1. Let *f* . If *f* is continuous and differentiable on , what is the value of *b*?

A. B. C. D. E.

1. If one side of a rectangle, *a*, is increasing at a rate of 3 inches per minute while the other side, *b*, is decreasing at a rate of 3 inches per minute, which of the following must be true about the area *A* of the rectangle?

A. *A* is always increasing B. *A* is always decreasing C. *A* is constant  
  
 D. *A* is decreasing only when E. *A* is decreasing only when

1. Compute .

A. B. 6C. D. E. 4

1. What is the maximum area of a rectangle inscribed in a semicircle of diameter 10 inches?

A. in2 B. 12.5 in2 C. in2 D. 25 in2 E. in2

1. Evaluate .

A. B. C. D. E. does not exist

1. Evaluate .

A. B. C. D. 1 E.

1. Evaluate

A. B. C. D. E. does not exist

1. What is the minimum value of ?

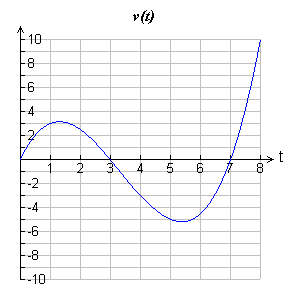
A. B. C. D. E. none of these

1. Evaluate .

A. B. C.  
  
D. E.

1. Which of the following is the equation of the tangent line to the curve at the point ?

A. B. C. D. E. none of these



1. The graph displayed to the right represents the velocity of an object moving horizontally on a straight path. Which of the following are true statements?
2. The object has stopped at *t* = 7.
3. The object is moving to the right on the interval .
4. The object is speeding up on the interval .
5. The object is slowing down on the interval .

A. i only B. i and ii C. ii and iv D. i and iii E. all of them

1. Evaluate

A. 2.5π B. 4 C. 5 D. 2π E. 6

1. Two cyclists leave the park at the same time with one traveling south at 9 mph and the other traveling west at 12 mph. How fast is the distance between the cyclists changing after 30 minutes?

A. 15 mph B. 21 mph C. 10.5 mph D. 20 mph E. 16.5 mph

1. Assume one worker can produce items at a rate of units per hour and a second worker produces items at a rate of , where *t* is time measured in hours. How many more units does the more productive worker produce in a typical 8 hour shift?

A. 97 units B. 63 units C. 27 units D. 16 units E. 12 units

1. A force of 100 N is used to stretch a spring from its natural length of 1 meter to a total length of 3 meters. What is the work done in stretching the spring?

A. 115 J B. 200 J C. 100 J D. – 100 J E. – 200 J

1. A rainstorm hit Portland, Maine, in October 1996, resulting in record rainfall. The rainfall rate *R*(*t*) on October 21 is recorded, in centimeters per hour, in the table below, where *t* is the number of hours since midnight. Use this information to determine the best estimate for the total rainfall during this 24 hour period.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *t* time in hours | 0 | 4 | 8 | 12 | 16 | 24 |
| *R*(*t*) in cm/hr | 0.5 | 0.3 | 1.0 | 2.5 | 1.5 | 0.6 |

A. 25.6 cm B. 26 cm C. 23.2 cm D. 27.6 cm E. 23.4 cm

1. A point in the region bounded by the curves , , and the *x*-axis is randomly chosen. Find the probability that the *x*-coordinate of the point is greater than 2.

A. B. C. D. E. none of the above

1. Let . Which of the following is true?

A. *g* is increasing on B. *g* is increasing on C. *g* is increasing on   
  
 D. *g* is decreasing on E. *g* is decreasing on

1. Evaluate

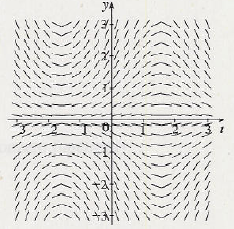
A. B. C. D. E. divergent

1. Evaluate .

A. B. C. D. E. divergent

1. Let *f* and *g* be differentiable functions of *x*. . If *g*(3) = 1 and , determine

A. B. C. D. E.



1. A certain first order differential equation has the direction field to the right. Which of the following could be that differential equation?

A. B. C. D. E.

1. From rest, a car accelerates over the course of 4 minutes, its velocity increasing quickly at first and then leveling off at 60 mph. It travels at 60 mph for 3 minutes, then decelerates steadily to 40 mph over the course of six minutes, at which point it decelerates to a stop over the course of 6 minutes, decelerating slowly at first and then more rapidly. Which of the following distances is a possible answer to the question, “How many miles has the car traveled?”

A. 12 B. C. 16 D. 18 E. 20

1. Which of the following integrals gives the length of the graph between *x* = *a* and *x* = *b*, where ?

A. B. C.   
  
D. E.

1. If the improper integral converges, then which of the following series must converge?

A. B. C. D. B and C E. All three series

1. Evaluate: 

A. B. C. D. E. none of these