***Second Annual NCMATYC Calculus Tournament***

***November 17, 2012***

***Morning Component***

***Good morning!***

***Please do NOT open this booklet until given the signal to begin.***

***There are 40 multiple choice questions and you will be given 90 minutes to complete the test. Answer the questions on the electronic grading form by giving the best answer to each question.***

***The scoring will be done by giving one point for each question answered correctly and zero points for each question answered incorrectly or left blank. Thus, it is to your advantage to answer as many questions as possible, even if you have to guess. If there is a tie, question number 35 will be used as a tie-breaker.***

***This test was designed to be a CHALLENGE. Do not waste time on questions you are unable to answer; focus and take pride in those questions which you ARE able to answer.***

***You may write in the test booklet. You may keep your test booklet and any of your scrap papers. Only the electronic grading form will be collected and graded.***

***Good luck!***

***Do Not Open Until Signaled.***

1. Let  and . Determine .

A.  B. 2 C.  D. 8 E. 4

1. Let , where . Which of the following is necessarily true?
2. 
3. 
4. 

A. III only B. II and III C. I and II D. I and III E. I, II, and III

1. Let  be a twice differentiable function and . Determine .

A.  B.  C.   
  
D.  E. undefined

1. Suppose and are continuous functions and  and . Determine .

A. 1.8 B. −3.5 C. 3.5 D. 1 E. −1

1. What is 35th derivative of the function ?

A.  B. C.D.  E. none of these

1. What is ?

A.  B.  C. 0 D.  E. does not exist

1. Let What value of  will make  a continuous function?

A.  B. C. D.  E. No such  exists.

1. Compute .

A.  B. C. D.  E. 

1. What is 

A. 0 B. C.1 D. 2 E. 

1. Let *f*(*x*) be a quadratic polynomial. Given that the minimum value of *f* occurs when  and , then what is the value of ?

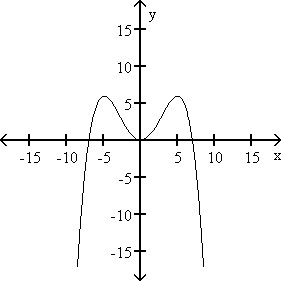
A. −8 B. −4C. D. 4 E. 8

1. Assume  is continuous and differentiable such that  and . What is the value of ?

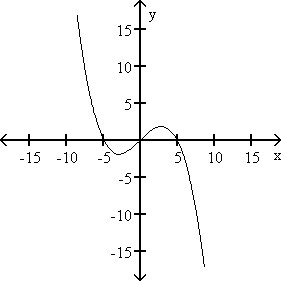
A. 32 B. 16 C. −16 D. −32 E. none of these

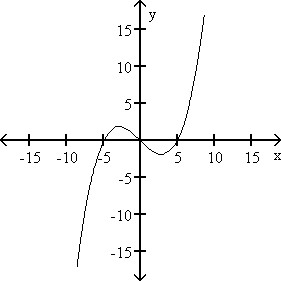
1. Evaluate .

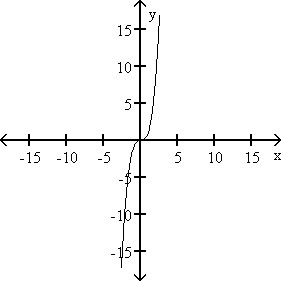
A. 3 B. 0C.  D.  E. 

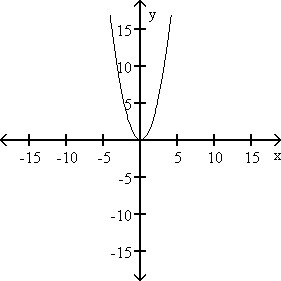


1. The graph of a function is given to the right. Which of the following represents the graph of its derivative?



1. B.

C. D.



E. None of the above

1. Determine the absolute minimum of  on the closed interval .

A.  B. C. **** D.  E. 

1. Let . If *f* is continuous on , what is the value of ?

A.  B. −3C.3 D. 5 E. none of these

1. Evaluate .

A. 3 B. 0C. 1 D. **** E. 

1. Compute .

A.  B. C. 0 D.  E. 

1. Which of the following must be true to ensure  has no local extema?

A.  B.  C.  D.  E. none of these

1. Evaluate .

A. 0 B. C.  D.  E. does not exist

1. Let . Which of the following is a true statement?
2.  represents the average value of  on  and has a maximum at .
3.  represents the average value of  on  and has a maximum at .
4.  represents the average value of  on  and has a minimum at .
5.  represents the average value of  on  and has a minimum at .
6. None of the above
7. Evaluate .

A.  B. 4C. 8 D.  E. does not exist

1. What is 6th derivative of the function ?

A.  B. C.D.  E. 

1. Which of the following is equivalent to ?

A.  B. C.  
  
D.  E. 

1. Evaluate 

A. 1 B. 2C. **** D.  E. does not exist

1. Determine the equation of the line with positive slope that is tangent to both  and .

A.  B. C.  D.  E. 

1. Let . Determine .

A. −4 B. 2C. 0 D. 3 E. does not exist

1. What is the probability that a randomly chosen point in the area enclosed by  and  has an **-value less than 0?

A.  B. C.  D.  E. 

1. Which of the following is equivalent to ?

A. 

B. 

C.

D. 

E. 

1. Determine  if .

A.  B.  C.  D.  E. 

1. What is the perimeter of the rectangle of largest area that can be inscribed in a semicircle of radius 3 inches?

A.  inches B.  inches C.  inches D.  inches E.  inches

1. Compute 

A.  B.  C. 0 D.  E. It is divergent.

1. Let  be the set of all values  such that  is a solution of . Which of the following is the sum of the elements of ?

A. −4 B. −5 C. 4 D. 5 E. −6

1. Determine the equation of the normal line to  at .

A.  B. C.  D.  E. none of these

1. Determine the arc length of  from  to .

A.  B. C.  D.  E. 

1. An open rectangular box is formed by cutting equal-size squares from the corners of a piece of cardboard 24 inches by 24 inches and folding up the sides. What is the ratio of the volume to the surface area for the box of maximal volume?

A. 1:1 B. 1:2 C. 2:1 D. 4:3 E. 3:4

1. Let *R* represent the region in quadrant I bounded by the *x*-axis, , , and the function . Determine the equation of the vertical line that divides the area of *R* into two equal-area regions.

A.  B.  C.  D.  E. 

1. Let *R* be the region bounded by  and . Let  be the volume of the solid of revolution formed by revolving *R* about the *x*-axis. Let  be the volume of the solid of revolution formed by revolving *R* about the *y*-axis. What is the ratio of  and ?

A. 1:1 B. 5:3 C. 3:5 D. 4:3 E. 2:1

1. Which of the following is the general solution for the differential equation ?

A.  B.  C.  D.  E. none of these

1. A vertical right circular cylindrical tank measures 24 feet high and 16 feet in diameter. It is full of oil weighing 60 lb/ft3. How much work does it take to pump the oil to a level 2 feet above the top of the tank?

A. 2,580,480π B. 1,290,240π C. 1,752,200π D. 1,105,923π E. none of these

1. Suppose a wire 27 inches long is cut into 2 pieces. One piece is made into a regular hexagon and the other piece is made into an equilateral triangle. How long should the piece made into a hexagon be in order to minimize the total area enclosed by the two pieces?

A. 10.8 inches B. 9 inches C. 13.5 inches D. 16.2 inches E. 18 inches