***Third Annual NCMATYC Math Competition***

***November 16, 2013***

***Calculus Test***

***Please do NOT open this booklet until given the signal to begin.***

***You have 90 minutes to complete this 40-question multiple choice calculus test. Approximately two-thirds of the questions pertain to topics covered in a typical Calculus I course; the rest pertain to topics covered in a typical Calculus II course. Answer the questions on the electronic grading form by giving the best answer to each question.***

***The scoring will be done by giving one point for each question answered correctly and zero points for each question answered incorrectly or left blank. Note that the scoring does not include a “guessing penalty” and so it is to your advantage to answer all the questions, even if you have to guess. If there is a tie, question number 19 will be used as a tie-breaker.***

***This test was designed to be a CHALLENGE. There will surely be questions that you will not be able to answer. We hope you will enjoy working on and take pride in those questions which you are able to answer.***

***You may write in the test booklet. You may keep your test booklet and any of your scrap papers. Only the electronic grading form will be collected and graded.***

***Good luck!***

1. Suppose and are differentiable functions for all real numbers and . Furthermore, it is known that . Determine .

A.  B. 16 C. 45 D.  E. 18

1. What is ?

A.  B.  C.  D. 0 E. does not exist

1. What is the 20th derivative of the function ?

A.  B.  C.   
  
D.  E. 

1. Let and be differentiable functions for all real numbers, what is ?

A.  B.  C.  D.  E. none of these

1. Which of the following types of functions must have a derivative equal to 0 at some value in its domain?

A. Cubic B. Quartic C. Exponential D. Tangent E. none of these

1. Let and be differentiable functions such that , , and . What is the value of ?

A. −12 B. −18 C. 20 D. 12 E. 0

1. What is ?

A.  B. undefined C.  D. 0 E. none of these

1. Determine  for .

A.  B. C. D.  E. 

1. Compute 

A.  B. C.  
  
D.  E. 

1. Which of the following is an inflection point on the graph of?

A. There is no inflection point B. C. D.  E. none of these

1. What is the equation of the tangent line to the graph of  at ?

A.  B. C. D.  E. none of these

1. Evaluate: 

A. e B. 2e C. e2 D. 0 E. 

1. For how many integer values of ***c*** does  equal a real number?

A. 1 B. 0C. 2 D. infinitely many E. impossible to determine

1. For how many values of the domain does the function  have a horizontal tangent line?

A. 0 B. 1 C. 2 D. 4 E. infinitely many

1. Which of the following is the derivative of the function ?
2. 
3. 
4. 

A. i B. iiC.iii D. none of these E. all of these

1. Evaluate 

A. 2 B. C. D.  E. none of these

1. Evaluate the first derivative of .

A.  B.   
  
D.  E. 

1. Define . Evaluate .

A.  B. C.   
  
D.  E. 

1. Compute the local extrema of  for .

A. No local extrema for  B.  C.  D.  E. none of these

1. Suppose the radius of a cylinder is measured at 2 cm with a possible error of ±0.01 cm and the height is measured at 5 cm with a possible error of ± 0.1 cm. Estimate the error in the calculated volume using differentials to the nearest hundredth of a cubic centimeter.

A. 2.2π cm3 B. 2.4π cm3C. 0.6π cm3 D. 0.8π cm3 E. 1.0π cm3

1. Let the quadratic function  be non-negative on the interval . Which of the following is ?
2.  B.  C.   
     
   D.  E. None of these
3. Which of the following is represents the volume of the solid formed by revolving the region bounded by the graphs of , *x*-axis, and the line  about the line .

A.  B. C.   
  
D.  E. None of these

1. The intersection of  and  creates two tangent lines, one to each curve. Which of the following expressions gives the angle created by the intersection of these two tangent lines?

A.  B. C.  
  
D.  E. 

1. ***L*** be the tangent line to  at the point . What is the sum of the intercepts of ***L***?

A.  B. C.D.  E. 

1. What is the arc length of  from  to ?

A.  B. C. D.  E. 

1. Determine the minimal distance from the point  to the graph .

A.  B. C.  D.  E. 

1. A circle centered at the origin passes through the point . Determine the area in Quadrant IV between the tangent line to the circle at  and the circle.

A.  B. C.  D.  E. 

1. Let R be the region in Quadrant I bounded by , the line , and the line . Calculate the volume of the solid of revolution generated by revolving R about the line .

A.  B. C.  D.  E. 

1. What is the value of ?

A. 1 B. 2C. 4 D. 6 E. 8

1. Determine  if

A.  B.  C.  D.  E. None of these

1. A particular radio station plays eight minutes of music followed by two minutes of commercials. If a listener tunes in randomly, what is the average number of minutes he will listen before he hears a commercial?

A. 3.2 min B. 4 min C. 2 min D. 2.8 min E. 3.6 min

1. Determine the area of the largest rectangle that can be inscribed in a semicircle of radius 4 cm.

A. 8 cm2 B. 16 cm2 C. 12 cm2 D. 10 cm2 E. 32 cm2

1. Determine the equation of the normal line to the curve  at .

A.  B. C.  D.  E. 

1. Evaluate .

A.  B. C.  D.  E. 

1. What is the length of the curve ?

A.  B. 1 C. 2 D.  E. none of these

1. Let  and . What is ?

A.  B.  C.  D.  E. none of these

1. Evaluate .

A. It is divergent. B.  C.  D.  E. 

1. Which of the following is the general solution for the differential equation ?

A.  B.  C.   
  
D.  E. 

1. A rope is 40 feet long and weighs 0.5 lb/ft. If it hangs over the edge of a building 200 feet tall, how much work is done by pulling half of the rope to the top of the building?

A. 240 ft-lb B. 300 ft-lb C. 260 ft-lb D. 380 ft-lb E. none of these

1. Find the centroid of the region bounded by the curve 4x2 – 24x + y2 + 10y + 45 = 0
2. (0, 0) B. (1, -3) C. (2, -4) D. (3, -5) E. (4, -6)