***Third Annual NCMATYC Calculus Tournament***

***November 22, 2014***

***Morning Component***

***Good morning!***

***Please do NOT open this booklet until given the signal to begin.***

***There are 40 multiple choice questions and you will be given 90 minutes to complete the test. Record your answers on the electronic grading form by giving the best answer to each question.***

***The scoring will be done by giving one point for each question answered correctly and zero points for each question answered incorrectly or left blank. Thus, it is to your advantage to answer as many questions as possible, even if you have to guess. If there is a tie, question number 24 will be used as a tie-breaker.***

***This test was designed to be a CHALLENGE. Do not waste time on questions you are unable to answer; focus and take pride in those questions which you ARE able to answer.***

***You may write in the test booklet. You may keep your test booklet and any of your scrap papers. Only the electronic grading form will be collected and graded.***

***Good luck!***

***Do Not Open Until Signaled.***

1. A company that manufactures digital cameras estimates that the profit, ***P***, in millions of dollars, can be modeled by for , where ***x*** is the amount spent on advertising in hundreds of thousands of dollars. Margaret computed the value of the instantaneous rate of change at  and found that . Which of the following is the correct interpretation of this value in context of the problem?

A. If the company spends $600,000 on advertising then the company is losing $840,000.

B. For the first $600,000 spent on advertising, the company loses $840,000.

C. When the company spends $600,000 on advertising, the instantaneous rate of change is decreasing at a rate of $840,000 per additional hundred thousand dollars.

D. When the company spends $600,000 on advertising, the profit is decreasing at a rate of $840,000 per hundred thousand dollars.

E. None of the above.

1. Suppose *f* and *g* are differentiable functions such that , , and . Compute the value of .

A. 2 B. 1 C. 0 D.  E. 

1. Determine the equation of the tangent line to at .

A.  B.  C.  D.  E. 

1. If , , , , and , compute .

A. 12 B. 8 C. 2 D. 1 E. none of these

1. What is the ?

A.  B. 0 C.  D.  E. undefined

1. Let  and . Compute .

A. 0 B.  C.  D. 1 E. 

1. At which of the following x-values will  have a tangent line whose slope is ?

A.  B. 0 C.  D.  E. none of these

1. Let  be a cubic function that has horizontal tangent lines at  and . What is the value of ?

A. 14 B. −7 C. 0 D. 7 E. −14

1. Compute: 

A.  B.  C.  D.  E. 

1. Which of the following curves has an inflection point but no minima or maxima? Assume .

A.  B.  C.  D.  E. none of these

1. How many of the lines listed below are tangent to the graph of ?

   

A. 0 B. 1 C. 2 D. 3 E. 4

1. Determine  for .

A.  B.  C.  D.  E. 

1. Suppose  is continuous on  and  and . Which of the following is implied by the Intermediate Value Theorem for continuous functions?

A. 

B. There is a unique *c* in , where .

C. There is a value *c* in , where  equals the circumference of a circle with radius .

D. There is a value c in between 0 and 1, where  equals 6 or .

E. None of the above

1. What is the minimum degree of an odd polynomial with at least two inflection points?

A. 4 B. 5 C. 7 D. 9 E. not enough information

1. If , compute .

A. 

B. 

C. 

D. 

E. none of the above

1. For what x-values is  non-differentiable?

A. 0 and  where k is any integer

B.  where k is any odd integer

C.  where k is any integer

D. 0 and  where k is any odd integer

E. The function is differentiable everywhere.

1. If  and  for , how small can  possibly be?

A. 16 B. 10 C. 8 D. 14 E. None of these

1. What is the maximum value for ?

A.  B.  C.  D.  E. 

1. What is the ?

A. undefined B. 0 C.  D. 1 E. 

1. If  and . Let . Evaluate .

A.  B.  C.  D.  E. 

1. What time could it be if the tip of the hour hand on a clock is in a position such that the ratio of the rate of change in its vertical position to the rate of change in its horizontal position is -?

I. 2:00 II. 5:00 III. 8:00

A. I only B. II only C. I and III D. I, II, and III E. none of these

1. What is the ?

A.  B.  C.  D.  E. 

1. A light in a lighthouse 1 km offshore from a straight shoreline is rotating at 2 revolutions per minute. How fast is the beam moving along the shoreline (in km/min) when it passes a point ½ km from the point on the shoreline closest to the lighthouse?

A.  B.  C.  D.  E. 

1. A wire of length 100 inches is cut into two pieces; one is bent to form a square, the other an equilateral triangle. If the cut is to be made such that the sum of the two areas is to be a minimum, find the perimeter of the square.

A.  B.  C.  D.  E. none of these

1. Let f(x) and g(x) be such that . Which of the following must be true?

A.  B.  C.  D.  E. 

1. What is the volume of the solid obtained when the triangle with vertices (1,0), (5,0), and (3,4) is rotated around the x-axis?

A.  B.  C.  D.  E. none of these

1. Evaluate .

A. 31 B. 35 C. 49 D. 51 E. 39

1. Calculate the area of an ellipse with major axis 10 units and minor axis 6 units.

A.  B.  C.  D.  E. 

1. 

A. 0 B.  C. 1 D. undefined E. none of these

1. =

A. 0 B.  C.  D.  E. 

1. Define  to be a periodic function on all reals with period two units such that  on the interval . Evaluate .

A. 0 B. 2 C. 5 D. 10 E. undefined

1. A rectangle with vertices of the form  is inscribed in the ellipse . Maximize the area of the inscribed rectangle.

A.  B.  C. 48 D. 96 E. 

1. Let R be the region bounded by , , and the x-axis. Let  be the volume of the solid of revolution formed by revolving R about the x-axis. Let  be the volume of the solid of revolution formed by revolving R about the y-axis. What is the ratio of  to ?

A. 1:3 B. 3:1 C. 3:2 D. 2:3 E. 1:

1. A cylinder of radius 5 feet and depth 10 feet is filled with oil. If the cost in dollars per cubic foot to pump out the oil is given by  where  is the depth of the oil in the cylinder, what is the total cost to empty the cylinder?

A. $200 B.  C. $5000 D.  E. 

1. A 50-foot chain that weighs 5 lbs/ft is hanging from a ledge. How much work is done in winding it up until its lowermost link is 25 feet from the ledge?

A. 4687.5 ft-lbs B. 3125 ft-lbs C. 3750 ft-lbs D. 1562.5 ft-lbs E. 7500 ft-lbs

1. If  is differentiable at , evaluate .

A.  B.  C.  D.  E. does not exist

1. Evaluate .

A.  B.  C. 

 D.  E. None of these

1. What is the derivative of  with respect to ?

A.  B.  C.  D.  E. 

1. A particle moves along the curve . Find the Cartesian coordinates of the point on the curve where the rate of change for the x and y coordinates of the particle is the same.

A.  B.  C.  D.  E. None of these

1. What is the solution of the differential equation ?

A.  B.  C.  D.  E. 